

2. A curve has equation

$$x^3 - 2xy - 4x + y^3 - 51 = 0.$$

Find an equation of the normal to the curve at the point (4, 3), giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(8)

4. The line l_1 has vector equation $\mathbf{r} = \begin{pmatrix} 11 \\ 5 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$, where λ is a parameter.

The line l_2 has vector equation $\mathbf{r} = \begin{pmatrix} 24 \\ 4 \\ 13 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ 1 \\ 5 \end{pmatrix}$, where μ is a parameter.

(a) Show that the lines l_1 and l_2 intersect. **(4)**

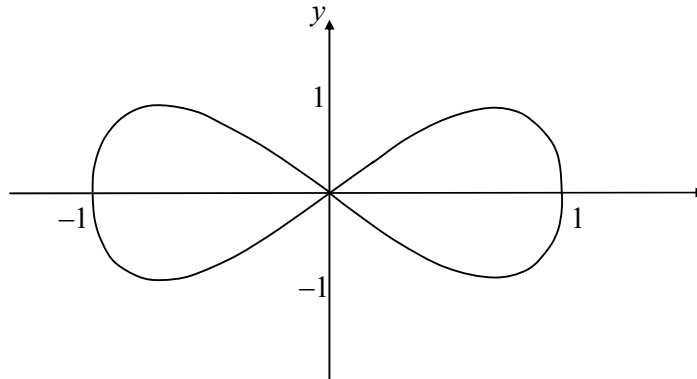
(b) Find the coordinates of their point of intersection. **(2)**

Given that θ is the acute angle between l_1 and l_2 ,

(c) find the value of $\cos \theta$. Give your answer in the form $k\sqrt{3}$, where k is a simplified fraction. **(4)**

5.

Figure 1



The curve shown in Fig. 1 has parametric equations

$$x = \cos t, \quad y = \sin 2t, \quad 0 \leq t < 2\pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of the parameter t . (3)

(b) Find the values of the parameter t at the points where $\frac{dy}{dx} = 0$. (3)

(c) Hence give the exact values of the coordinates of the points on the curve where the tangents are parallel to the x -axis. (2)

(d) Show that a cartesian equation for the part of the curve where $0 \leq t < \pi$ is

$$y = 2x\sqrt{1-x^2}. \quad (3)$$

(e) Write down a cartesian equation for the part of the curve where $\pi \leq t < 2\pi$. (1)

6.

Figure 2

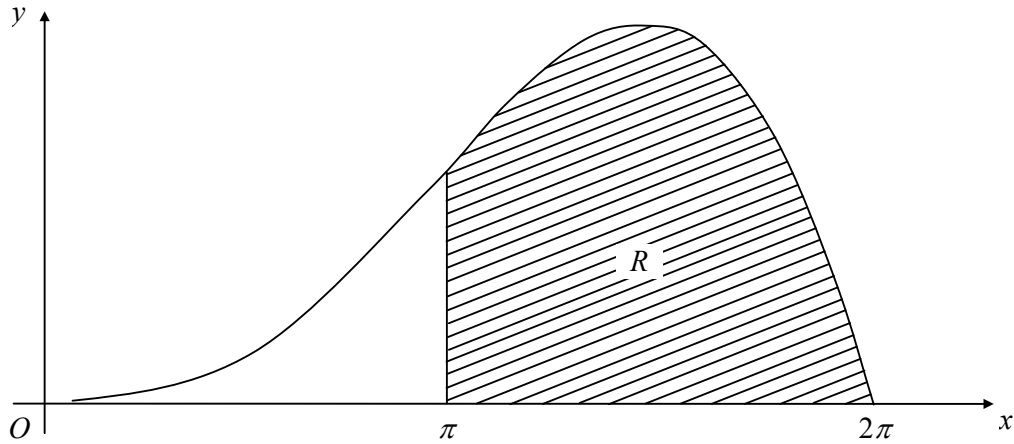


Figure 2 shows the curve with equation

$$y = x^2 \sin\left(\frac{1}{2}x\right), \quad 0 < x \leq 2\pi.$$

The finite region R bounded by the line $x = \pi$, the x -axis, and the curve is shown shaded in Fig 2.

- (a) Find the exact value of the area of R , by integration. Give your answer in terms of π . (7)

The table shows corresponding values of x and y .

x	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y	9.8696	14.247	15.702	G	0

- (b) Find the value of G . (1)

- (c) Use the trapezium rule with values of $x^2 \sin\left(\frac{1}{2}x\right)$

- (i) at $x = \pi$, $x = \frac{3\pi}{2}$ and $x = 2\pi$ to find an approximate value for the area R , giving your answer to 4 significant figures,

- (ii) at $x = \pi$, $x = \frac{5\pi}{4}$, $x = \frac{3\pi}{2}$, $x = \frac{7\pi}{4}$ and $x = 2\pi$ to find an improved approximation for the area R , giving your answer to 4 significant figures. (5)

7. In an experiment a scientist considered the loss of mass of a collection of picked leaves. The mass M grams of a single leaf was measured at times t days after the leaf was picked.

The scientist attempted to find a relationship between M and t . In a preliminary model she assumed that the rate of loss of mass was proportional to the mass M grams of the leaf.

(a) Write down a differential equation for the rate of change of mass of the leaf, using this model. (2)

(b) Show, by differentiation, that $M = 10(0.98)^t$ satisfies this differential equation. (2)

Further studies implied that the mass M grams of a certain leaf satisfied a modified differential equation

$$10 \frac{dM}{dt} = -k(10M - 1), \quad (I)$$

where k is a positive constant and $t \geq 0$.

Given that the mass of this leaf at time $t = 0$ is 10 grams, and that its mass at time $t = 10$ is 8.5 grams,

(c) solve the modified differential equation (I) to find the mass of this leaf at time $t = 15$. (9)
