
 Core Mathematics C4 Advanced Level Mock Paper

Time: 1 hour 30 minutes

## Materials required for examination <br> Items included with question papers Mathematical Formulae Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

## Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

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Turn over

1. Use the substitution $u=4+3 x^{2}$ to find the exact value of

$$
\int_{0}^{2} \frac{2 x}{\left(4+3 x^{2}\right)^{2}} \mathrm{~d} x
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2. A curve has equation

$$
x^{3}-2 x y-4 x+y^{3}-51=0 .
$$

Find an equation of the normal to the curve at the point (4, 3), giving your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
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3.

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\mathrm{f}(x)=\frac{1+14 x}{(1-x)(1+2 x)}, \quad|x|<\frac{1}{2} .
$$

(a) Express $\mathrm{f}(x)$ in partial fractions.
(b) Hence find the exact value of $\int_{\frac{1}{6}}^{\frac{1}{3}} \mathrm{f}(x) \mathrm{d} x$, giving your answer in the form $\ln p$, where $p$ is rational.
(c) Use the binomial theorem to expand $\mathrm{f}(x)$ in ascending powers of $x$, up to and including the term in $x^{3}$, simplifying each term.
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4. The line $l_{1}$ has vector equation $\mathbf{r}=\left(\begin{array}{r}11 \\ 5 \\ 6\end{array}\right)+\lambda\left(\begin{array}{l}4 \\ 2 \\ 4\end{array}\right)$, where $\lambda$ is a parameter.

The line $l_{2}$ has vector equation $\mathbf{r}=\left(\begin{array}{c}24 \\ 4 \\ 13\end{array}\right)+\mu\left(\begin{array}{l}7 \\ 1 \\ 5\end{array}\right)$, where $\mu$ is a parameter.
(a) Show that the lines $l_{1}$ and $l_{2}$ intersect.
(b) Find the coordinates of their point of intersection.

Given that $\theta$ is the acute angle between $l_{1}$ and $l_{2}$,
(c) find the value of $\cos \theta$. Give your answer in the form $k \sqrt{ } 3$, where $k$ is a simplified fraction.
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The curve shown in Fig. 1 has parametric equations

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x=\cos t, y=\sin 2 t, \quad 0 \leq t<2 \pi .
$$

(a) Find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of the parameter $t$.
(b) Find the values of the parameter $t$ at the points where $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$.
(c) Hence give the exact values of the coordinates of the points on the curve where the tangents are parallel to the $x$-axis.
(d) Show that a cartesian equation for the part of the curve where $0 \leq t<\pi$ is

$$
y=2 x \sqrt{ }\left(1-x^{2}\right)
$$

(e) Write down a cartesian equation for the part of the curve where $\pi \leq t<2 \pi$.
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Figure 2 shows the curve with equation

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y=x^{2} \sin \left(\frac{1}{2} x\right), \quad 0<x \leq 2 \pi .
$$

The finite region $R$ bounded by the line $x=\pi$, the $x$-axis, and the curve is shown shaded in Fig 2.
(a) Find the exact value of the area of $R$, by integration. Give your answer in terms of $\pi$.

The table shows corresponding values of $x$ and $y$.

| $x$ | $\pi$ | $\frac{5 \pi}{4}$ | $\frac{3 \pi}{2}$ | $\frac{7 \pi}{4}$ | $2 \pi$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 9.8696 | 14.247 | 15.702 | $G$ | 0 |

(b) Find the value of $G$.
(c) Use the trapezium rule with values of $x^{2} \sin \left(\frac{1}{2} x\right)$
(i) at $x=\pi, x=\frac{3 \pi}{2}$ and $x=2 \pi$ to find an approximate value for the area $R$, giving your answer to 4 significant figures,
(ii) at $x=\pi, x=\frac{5 \pi}{4}, x=\frac{3 \pi}{2}, x=\frac{7 \pi}{4}$ and $x=2 \pi$ to find an improved approximation for the area $R$, giving your answer to 4 significant figures.
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7. In an experiment a scientist considered the loss of mass of a collection of picked leaves. The mass $M$ grams of a single leaf was measured at times $t$ days after the leaf was picked.

The scientist attempted to find a relationship between $M$ and $t$. In a preliminary model she assumed that the rate of loss of mass was proportional to the mass $M$ grams of the leaf.
(a) Write down a differential equation for the rate of change of mass of the leaf, using this model.
(b) Show, by differentiation, that $M=10(0.98)^{t}$ satisfies this differential equation.

Further studies implied that the mass $M$ grams of a certain leaf satisfied a modified differential equation

$$
\begin{equation*}
10 \frac{\mathrm{~d} M}{\mathrm{~d} t}=-k(10 M-1) \tag{I}
\end{equation*}
$$

where $k$ is a positive constant and $t \geq 0$.
Given that the mass of this leaf at time $t=0$ is 10 grams, and that its mass at time $t=10$ is 8.5 grams,
(c) solve the modified differential equation (I) to find the mass of this leaf at time $t=15$.
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